**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans :- The probability of that service manager cannot meet is commitment that is**

1. **0.2676**

**Explaination :- The service manager work begin on the transmission of a customer’s car 10 minutes after. The time required for servicing transmissions is usualiy 45 min.**

**for normal distribution :-X=45, μ=55, б=8**

**z = (X-μ)/б**

**= (60-55)/8**

**= 0.625**

**In R software for probability finding we use function called pnorm.**

**As we want to find the probability of service manager cannot meet his commitment, So we should write below command.**

**1-pnorm(0.625)**

**=0.2659**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

****Ans :**The current age (in years) of 400 clerical employees at an insurance claims processing center mean = 38 & SD = 6**

**SD = 6**

**Z score = (Value - Mean)/SD**

**Z score for 44  = (44 - 38)/6  = 1  =>  84.13 %**

**=> People above 44 age = 100 - 84.13 =  15.87%  ≈  63    out of 400**

**Z score for 38  = (38 - 38)/6 = 0 => 50%**

**Hence People between 38 & 44  age = 84.13 - 50 = 34.13 % ≈  137 out of 400**

**Hence More employees at the processing center are older than 44 than between 38 and 44. i.e This is statement is F**ALSE.****

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

****Ans :-****

****M**ean = 38,SD = 6,**X= 30****

Z score for 30  = (30 - 38)/6 =  -1.33  =  9.15  %   ≈ 36 out of 400

****The finding of Z score is estimate that a** training program for employees under the age of 30 at the center would be attract near about 36 emp out of 400.i.e That statement is True.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:-**

**The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.**

**The Difference between 2X1 and (X1 + X2) is the magnitude they hold of two different sample subsets (X1 and X2) from the same source(population). X1 and X2 can be a different subset of a sample from a similar source (population) but If X1 ~ N(μ, σ2) then, 2 X1 ~ N(2 μ, 4 σ2 ) If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are iid normal random variables then (X1 + X2) ~N(2 μ, 2 σ2) . Hence, 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ2 + 2σ2 ) The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters.**

**The Normal distribution has two parameters, the mean, µ, and the variance, σ2. µ and σ2satisfy −∞ < µ < ∞, σ2> 0. We write X ∼ Normal (µ, σ2) or X ∼ N(µ, σ2 )**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:-** p(a<x<b) = 0.99 ,mean =100,SD = 20

**Identify symmetric values for the standard normal distribution such that the area enclosed is 0.99**

**From the above details,we have to excluded area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values**

**Using Python**

**Z value is given as stats.norm.ppf(pvalue)**

**Z value at 0.5th percentile is given as**

**Z(0.5) = stats.norm.ppf(0.005)= -2.576**

**Z value at 99.5 percentile is given as**

**Z(99.5) = stats.norm.ppf(0.995) = 2.576**

**Z = (x - 100)/20 = > x = 20z+100**

**a = -(20\*2.576) + 100= 48.5**

**b = (20\*2.576)+100= 151.5**

**Two values symmetric about mean for the given standard normal distribution are[48.5,151.5]**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Ans:-**

A):

95% of the ****probability**** ****lies****between 1.96 ****standard deviations****of the ****mean****.

Thus ****range****is:

= (12- 1.96 \* 5, 12 + 1.96 \* 5)

= ($ 2.2 M, $22 M)

= ($ 99 M, $1026 M)

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans:- p(Z** ≤ **p-12/5) =0.05**

From p ****values****of z ****score**** ****table****, we get:

P-12/5= -1.644

P=12-8.22=3.78

Thus at $3.78M ****dollars****, or Rs. 170.1M ****amount****, 5th ****percentile****of ****profit**** ****lies****.

Or 5th ****percentile****of ****profit****is Rs. 170.1 Million.

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:- 2nd Division**